Optimizing End Of Quarter Shot-Timing In The NBA: "Everyone Knows About The 2 For 1, But What About The 3 For 2?"

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Abstract

Since the advent of the shot clock, the "2-for-1" has become a common end of quarter strategy in the NBA. With this approach, a team will strategically time their shot in hopes of ensuring a second possession while limiting their opponent to a single possession. Prior research has shown the effectiveness of the "2-for-1" strategy but no well-known public study has explored extending this strategy to "3-for-2" or beyond. This paper summarizes a study which: (1) analyzes the effects that possession timing has on behavior as well as outcome; (2) quantifies the cost-benefit tradeoff of strategically "timing a possession;" and (3) proposes the optimal possession timing strategy to maximize expected points (as opposed to simply possessions). The research reveals how to improve end of quarter behavior in the NBA by better understanding the math behind why, and when, a "2-for-1" is beneficial and suggests how to extend this further to a "3-for-2".

Introduction

The average value of a possession in the NBA is estimated at just over 1 point [1]. If a team is able to capture an extra possession in only half of those quarters, this could mean the difference between a team winning a game and potentially making the playoffs. The 2013-2014 Phoenix Suns are an example of a team where extra possessions could have made a big difference. The Suns had a record of 2-9 in games decided by 3 points or less and they missed the playoffs by a single game [2]. With four end-of-quarter opportunities in a NBA game there is potential for strategic optimization. Using strategy to manipulate possessions and timing for end of quarter situations is not a novel concept. Prior research has been done on factors such as if, and when, to foul in late game situations [3] [4]. While most would agree that a "2-for-1" shot is good strategy it does not necessarily mean that a team should do everything in its power to achieve a "2-for-1." Although shooting a half-court shot with 30 seconds left will likely accomplish a "2-for-1," this is not considered a rational strategy. However, in a scenario with 32 seconds left and the options of: (1) pushing the pace for a quick shot (in hopes of achieving a "2-for-1"); or (2) slowing the pace for a less rushed shot the optimal strategy is not as clear. Not all "2-for-1" opportunities are the same as there are constraints at play, such as; how quickly an efficient shot can be attained.

While previous research has focused strictly on the "2-for-1" strategy, this paper proposes a new idea: there might be opportunities for a "3-for-2" strategy. In such a tactic a team might strategically use "possession timing" to better set themselves up for a more optimal "2-for-1". Rather than having to decide if it is worthwhile to force a shot with 30 seconds remaining, a team might avoid this question altogether by calmly running down the clock on a prior possession to achieve a more optimal "2-for-1" possession later on. Similarly, this concept could apply to a team who is sensing that they are on the wrong side of the "2-for-1." In theory, a team could use a counter strategy to force their opponent into choosing between a rushed "2-for-1" possession or giving up a possession advantage entirely.

This paper will attempt to quantify the comparative value between different timing based strategies. The research presented in this paper proves that there is more to consider in a "2-for-1" strategy than simply gaining an extra possession, as not all possessions are of equal value. This is done by introducing the concepts of Net Possession Advantage (NPA) and Net Point Differential (NPD). By analyzing the effects of existing "possession timing" based strategies, such as end-of-quarter and rushed "2-for-1" possessions, the value of a timed possession can be quantified. From the results, four different types of strategically "timed" possession types are proposed: desperation, rushed, timed, and timed desperation. Using this information, the study seeks to find an optimal strategy "cheat sheet" similar to that which NFL coaches use on the sidelines when deciding whether or not to go for a two-point conversion. Several different methodologies for defining an optimal strategy are proposed and explored including: simulating what would have been optimal historically, as well as simulating a world in which all teams are considering this type of research.

Methodology

Using 8 years of publically available NBA play-by-play data from the 2006-2013 seasons[1], the sequence of events for every game was reconstructed. This data was then translated into a list of possessions with specific attributes about each possession. 10,228 different games translated to 1,797,969 different possessions.

Definitions

- Possession start time: A possession is defined as starting when: the quarter begins, the opponent makes a shot or free throw which ends possession, the opponent commits a turnover, or when the team who is not already in control of a possession wins a jump ball or achieves a rebound.

[1] http://downloads.nbastuffer.com/nba-play-by-play-data-sets

- Possession end event: The first occurrence of a shot, free throw, or turnover[2] after a possession begins[3]
- Possession event time: The time of the first possession end event in a possession[4]
- Possession length: The difference between possession start time and possession event time
- Possession points: This includes all points scored by a team from possession start until the next team starts a possession[5]
- Shot clock remaining: The estimated shot clock remaining when a possession end event occurs following the rules defined in the NBA rule book[6] [5]
- Net point differential: The net point differential (difference in score between the teams) from the beginning of the following possession through the remainder of the quarter[7]
- Net possession advantage: The net possession advantage through the remainder of the quarter for the team currently in possession inclusive of the current possession[8]

Possession Timing Effects

Net Possession Advantage

The fundamental assumption of the "2-for-1" strategy is that if a team is able to time their second to last possession in the quarter properly, then an extra possession will be gained at the end of the quarter relative to their opponent. In order to understand the type of timing necessary to gain the "2-for-1," a concept called Net Possession Advantage (NPA) in introduced. NPA represents the number of "extra" possessions that a team will obtain through the remainder of the current quarter (inclusive of the current possession). On a single possession, this value would either be 0 or 1 depending on whether an extra possession is gained. Each second in a quarter has an associated NPA value, which is calculated by averaging single NPA values across all possessions analyzed in a dataset. Figure 1 shows how the NPA value changes over the course of the final 4 minutes of a quarter[9].

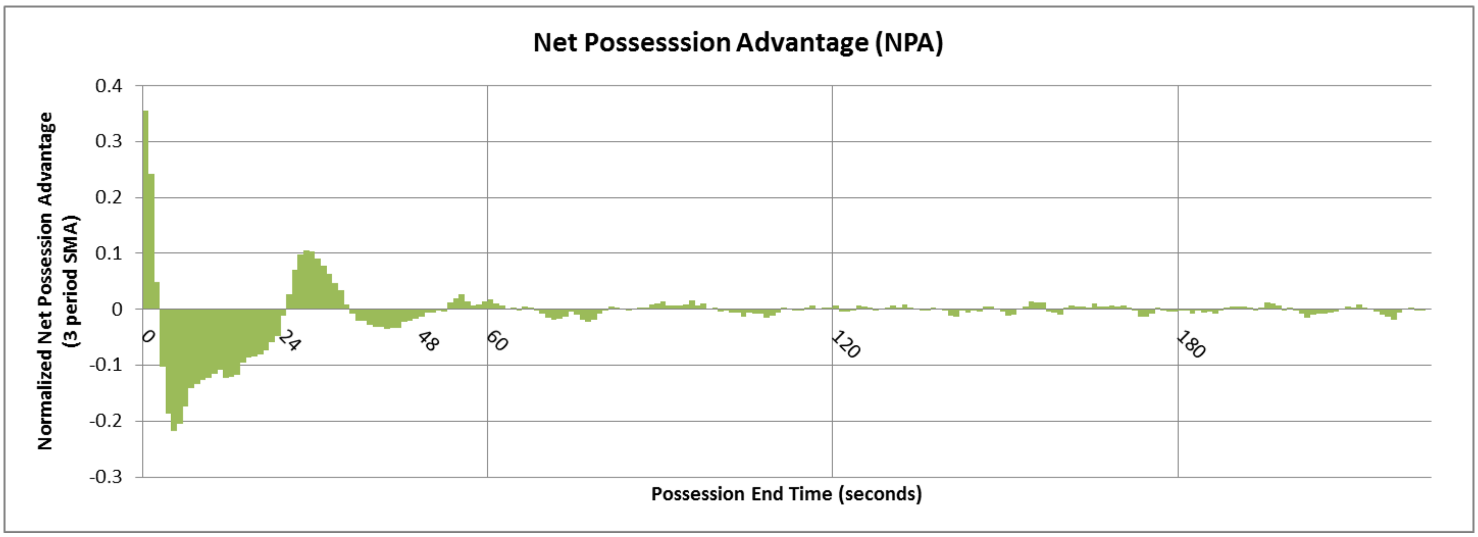

Figure 1. Net Possession Advantage

As seen in Figure 1, if a team ends a possession in the final 24 seconds (when the shot clock is shut off) then it is highly likely that the team will be at a possession disadvantage, because the opponent can use the remainder of the clock and not leave any time left. This does not necessarily hold true when ending a possession in the final 3 seconds, as the opposing team may not have time to take a shot. As expected, a possession ending before the 24 second mark will generally result in a "2-for-1" advantage. The positive "2-for-1" advantage window is seen at 25 seconds

[2] This includes offensive fouls.
[3] Possessions which ended with the end of the quarter were intentionally not captured.
[4] Offensive rebounds and subsequent shots/turnovers/etc. are not captured as a "new" possession and instead are added to the existing possession; however the event time is not extended for the possession to include those events.
[5] Offensive rebounds and subsequent points will be included in the total possession points.
[6] This includes logic for resetting the shot clock to 14 seconds when a foul occurs with the shot clock lower than 14 seconds. However the logic is not perfect as there are cases which cannot be interpreted from play-by-play data such as: if a foul occurs in the frontcourt vs. backcourt, 3 second violations, kicked balls, differentiation between foul types, and various jump ball scenarios.
[7] This would not include the points scored in the current possession including offensive rebounds resulting in additional points but would include all subsequent possessions in the period.
[8] Offensive rebounds and subsequent shots would still be included as part of this possession and would not count towards a possession advantage. In addition, only possessions which end in a possession end event would count as a possession. Therefore if the defense gains control of the ball via a rebound at the end of the quarter but fails to achieve an end of possession event (shot, turnover, etc.) before the end of the quarter this would not count towards a possession advantage whereas a full court desperation shot would.
[9] The values shown in the figure are normalized by subtracting 0.5 from them to better show when one team is in advantage compared to the other. A three period simple moving average is used to smooth out the graph as well as to account for improper time stamps for certain seconds.

through 35 seconds, with a peak at 28 seconds. Note that a full court shot as time expires does count as a possession, even if it offers relatively low value. Therefore, the disadvantage of the NPA metric is that it does not account for the value of a particular possession and is only accounting for raw number of possessions.

As figure 1 shows, prior to the "2-for-1" peak there are gradually decreasing peaks and valleys throughout most of the final 4 minutes. This implies that there might be opportunities to improve on the chances of a "2-for-1" by trying to manipulate timing to shift into one of the prior peak zones. The largest of these peaks can be found in the "3-for-2" range with a max value at 55 seconds and another lesser spike at 60 seconds[10]. The minimum trailing valleys seen in the figure occur at times of 1:11 and 1:17.

Net Point Differential

As noted, the NPA metric does not account for the value of a possession, or how many points these "advantages" actually translate to. To overcome such deficiencies a new metric called the Net Point Differential (NPD) is proposed which represents the net point differential for a team immediately following the end of their possession through the remainder of the quarter. By not including points from the current possession, this metric shows the value of subsequent possessions disregarding the quality of the current possession[11]. While NPD is a simple concept it can be a valuable metric as it naturally factors in many complexities such as: offensive rebounds, fouls, free throws, turnovers, timeouts, etc. This concept is similar to the plus/minus statistic in which the full impact of a player can be measured without needing to understand and factor in every piece of information about why a player might be providing value when on the court. These types of statistics quantify something or someone in terms of points, which is the most basic and important unit of measurement in the basketball. However, the downside of such comprehensive metrics is that the "why" behind the number may not be clear.

In Figure 2, the values of NPA and NPD are compared over the last 90 seconds of the first 3 quarters[12]. Figure 2 shows that NPD shows similar trends to NPA except for the peaks and valleys are shifted towards the end of the quarter. Rather than NPA improving with roughly 4 seconds remaining in the quarter, NPD starts to dramatically improve with 10 seconds remaining. This discrepancy is due to end of quarter "buzzer beater" shots, which count against NPA, but provide little impact on NPD. While the NPA has a peak "2-for-1" time at 28 seconds, the NPD peaks around 33 seconds. The NPD "2-for-1" window starts and ends later than NPA, and dramatically drops off at a much steeper rate than NPA (at approximately 39 seconds). The secondary peaks and valleys observed in NPA beginning at approximately 55 seconds do appear in NPD but they do not to consistently align with NPA. NPD may also be more volatile at these later times which could be explained by the point differential being more random the further you get from the end of the quarter.

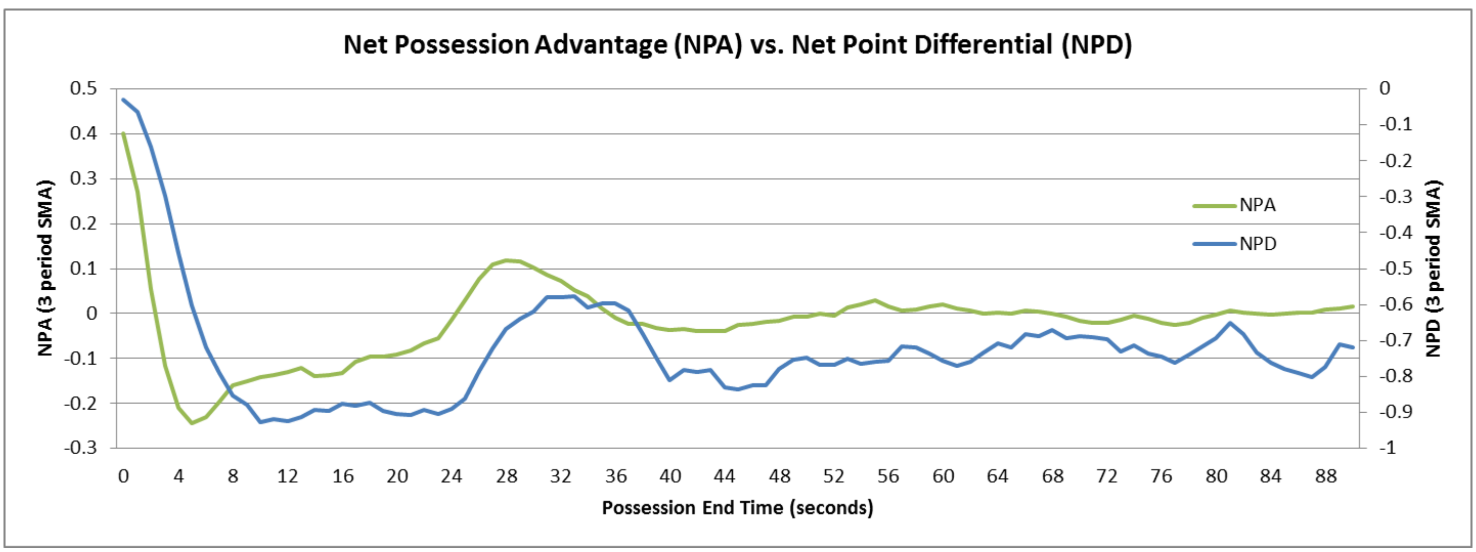

Figure 2. Net Possession Advantage (NPA) vs. Net Point Differential (NPD)

The most interesting takeaway from comparing NPA and NPD is that the optimal strategy is different if you are optimizing for possession as opposed to points. One logical explanation for this is that there are actually bad "2-for-1" possessions. For example, a possession ending with 24 seconds remaining may result in little value from an extra possession due to both that possession and their final possession being rushed. In addition, a rushed possession might actually be more likely to translate into a turnover and easy fast break points for the opponent hurting the team even more. The NPD metric will have these kinds of factors built into it whereas the NPA metric will not.

[10] At 60 seconds is where the data starts to have consistently shifted data points.
[11] This will also cause the NPD values to be consistently negative.
[12] The 4th quarter and overtime are not included because the dynamic of the game is sometimes different in those situations from things like blowouts, bench players playing, fouling, timeouts, etc.

Behavioral Changes

The NPD results clearly show that there are opportunities for gaining some sort of advantage toward the end of the quarter. Now to evaluate if and when teams are taking advantage of this by changing their behavior, behavioral changes are analyzed. To do so, the length of possessions can be compared across various times in the game. In Figure 3, the possession length distribution is compared for different possession start times.

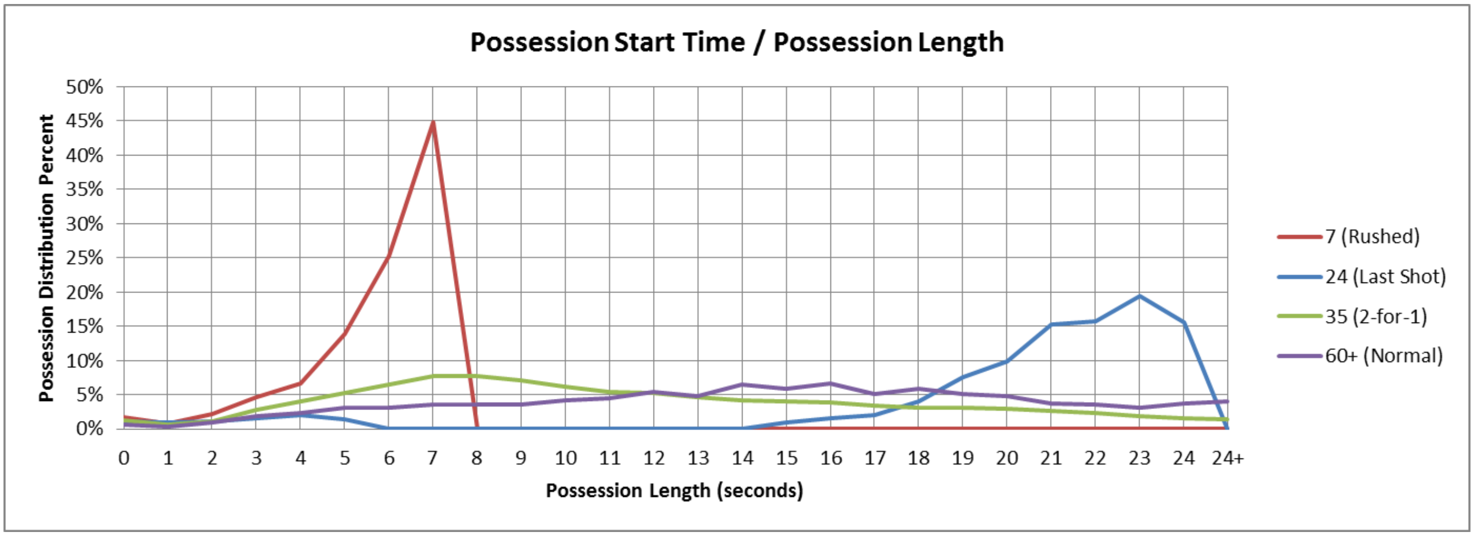

Figure 3. Possession Start Time / Possession Length

From the four different possession start times evaluated, it is very clear that there are behavioral changes occurring in certain situations. To gain a better understanding of when changes in behavior occur, the possession length can be compared across every second in the game. If behavioral changes are occurring, "normal" possession behavior must first be defined. It is conservatively estimated that the times when behavior may be influenced by strategic possession timing includes; the final two minutes of all quarters, the second half of the fourth quarter, or any part of overtime. There are other times when behavior may be slightly altered from other non-strategic timing based effects, such as the beginning of the quarter or when bench players typically come in. For this study, "normal behavior" is defined as all times, other than the whole fourth quarter, overtime, or the final two minutes of each quarter. With the remaining possession data the distribution of possession lengths can be calculated similar to that shown in Figure 3.

To evaluate when behavior changes occur throughout the game, a similar distribution for all possessions can be built and grouped by the second at which the possession started. Then using Pearson product-moment correlation[13] the per-second distributions can be compared with the "normal behavior" distribution. These correlation coefficients are plotted over time in Figure 4, where it is observed that after the first 30 seconds of the quarter these correlation coefficients are consistently above a correlation coefficient of 0.8 up to the final minute. At which point, the correlation values drop dramatically. There are two spikes at around 17 and 33 seconds where the values jump up near "normal behavior." When starting a possession with 33 seconds, a team may not feel there is enough time to perform a "2-for-1" and therefore they might be more likely to show "normal" behavior. The spike at 17 seconds might instead be more of a coincidence than anything, as the amount of time remaining is similar to the most frequent "normal" possession lengths.

[13] http://en.wikipedia.org/wiki/Pearson_product-moment_correlation_coefficient

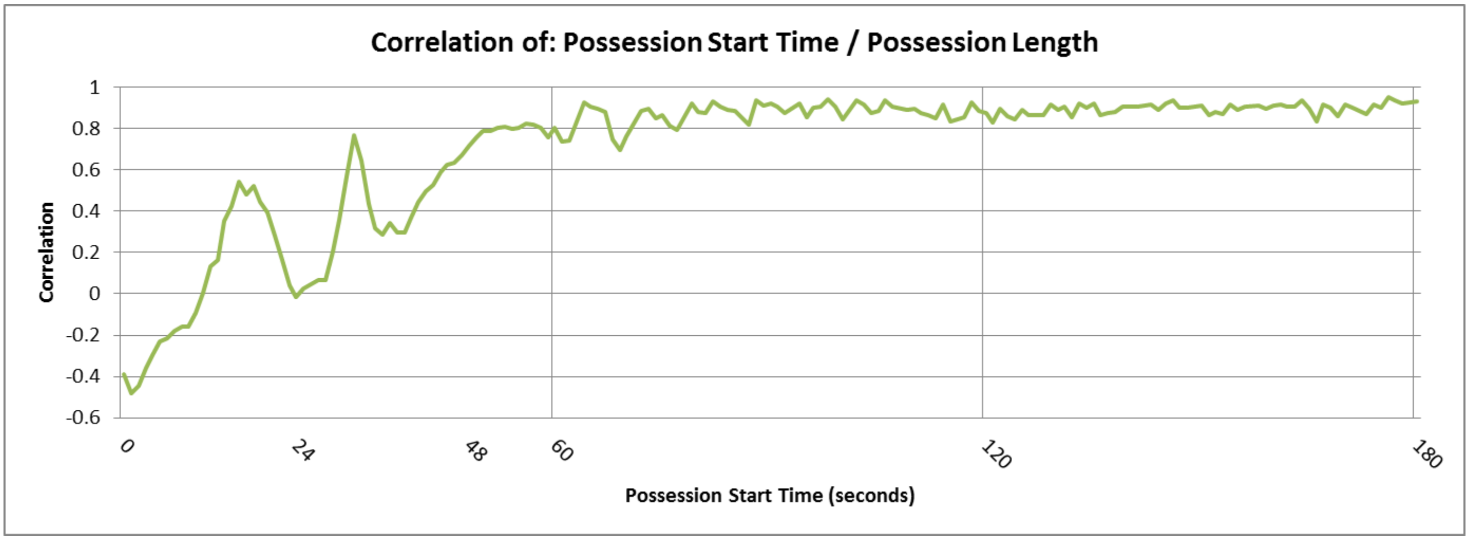

Figure 4. Correlation of: Possession Start Time / Possession Length

It is clear that teams are changing their behavior in certain situations; therefore, it is worthwhile to understand the intent of such behavioral changes. Team's intentions can be learned by looking at the distribution of when possessions are ending in the last two minutes. In figure 5, which shows the distribution of possession end times, the two most obvious increases occur at the end of the quarter and in the "2-for-1" timeframe. The reason for the end of the quarter possession end time is obvious as teams have no downside of taking a last second shot assuming they are optimizing for points, as opposed to field goal percentage. The "2-for-1" is also expected given the expected value of an extra possession. The most frequent time at which teams will complete their "2-for-1" possession falls in the range of 29 to 31 seconds. It seems that teams are performing their "2-for-1" possessions slightly after the 28 second peak seen for NPA, and just before the 33 second peak observed for NPD. There are a few minor blips in the data starting at around 44 seconds, none of which appear to be statistically significant, however it might be worthwhile to look more closely, to see if trends have changed at all in the last few years where analytics have become more prevalent. The correlation change observed with possessions ending in the 40 to 60 second range could somehow be contributed to the pace slowing down in the final minute to get a good shot, but this was not further explored. It does not appear as a whole that the NBA has been intentionally doing any strategic timing based strategies beyond the "2-for-1," but further research should be done broken down by year and team to confirm.

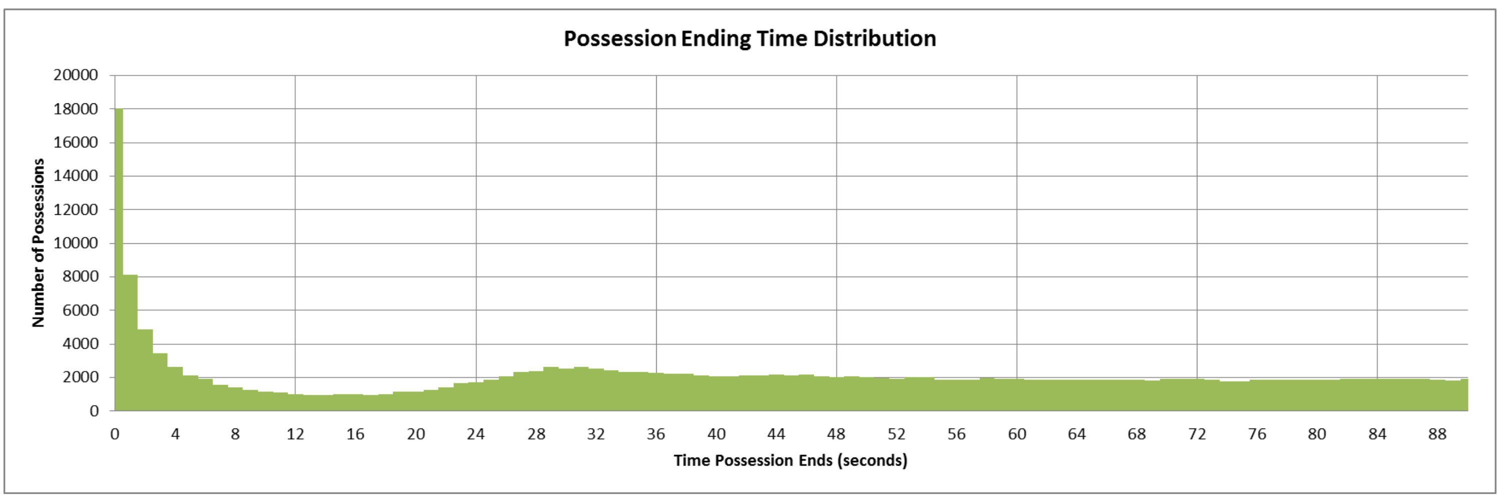

Figure 5. Possession Ending Time Distribution

Quantifying Value of a Timed Possession

A "timed possession" is a possession where a team will intentionally attempt to end the possession at a specific time in hopes of gaining some sort of advantage. This could be in the form of an extra possession via a "2-for-1" opportunity, or at the end of a quarter where maintaining possession when the clock hits zero is useless[14].

[14] This may not be true in the 4th quarter where teams might be running the clock out in a blow out as the risk of injury is not worth the benefit of scoring more points.

Does situational timing impact possession value?

As the research has shown, situational timing plays a role in influencing possession behavior. This was most notable in the final minute of each quarter. In order to properly evaluate the expected value of a timed possession as opposed to a normal possession, one must first understand if these behavioral changes have any impact on possession efficiency. If a team is changing their behavior, it could have a non-trivial impact on the outcome of the possession. To determine if such relationships exist between timing behavior and expected value, a metric called average points per possession (APPP) is calculated across every possible second in a quarter. In Figure 6 it is shown that the APPP values are consistent across the length of the quarter except for some variance at the beginning and ends of each quarter. The fluctuation at the start of the quarter is partially due to fewer samples, and is also likely caused by teams "settling in" at the beginning of a new quarter[15]. The changes in the last minute of the quarter correspond with the previously noted changes in possession timing. These changes in value are most dramatic in the "2-for-1" range as well as at the very end of quarter. These times align with when possession timing differed the most from the "normal" possession timing. Overall it appears as though the behavior changes are having an impact on value.

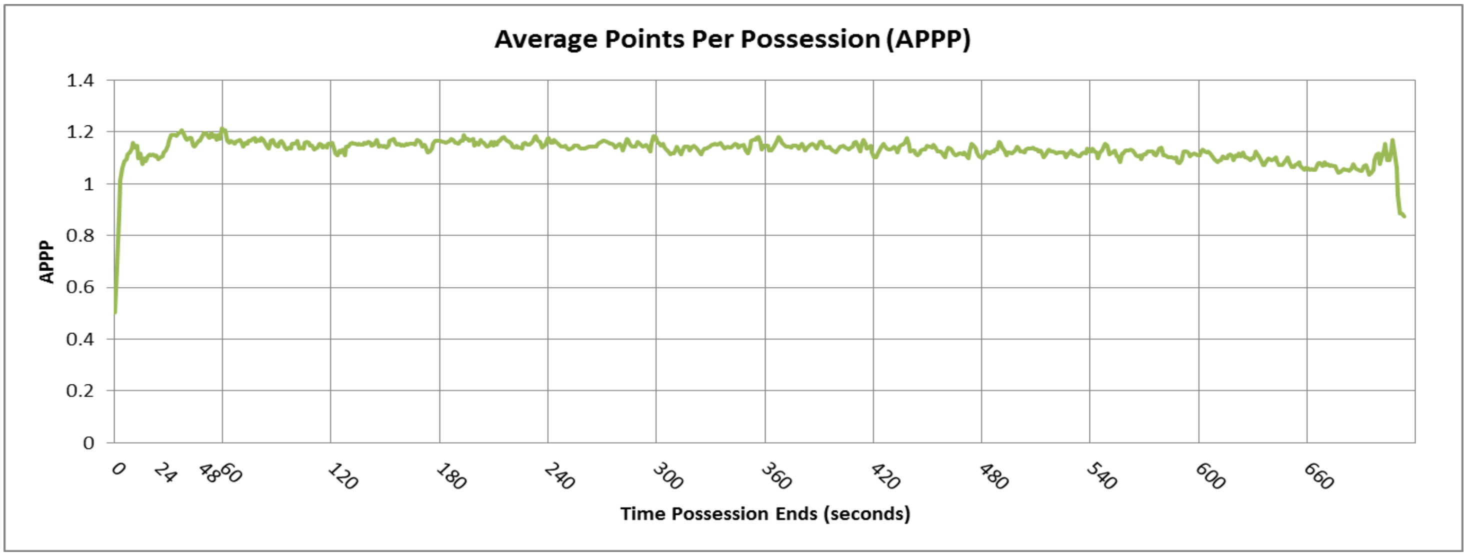

Figure 6. Average Points Per Possession

Are all timed possessions equal?

Existing research in "Tick Tock Shot Clock: Optimal Stopping in NBA Basketball" [6] has explored the dilemma of "if a shot opportunity should be taken or passed up in favor of continuing the possession" [6]. In the research it has been shown that possession efficiency decreases as the time remaining on the shot clock also decreases. During a timed possession, a team will artificially constrain its possession length in an attempt to end the possession at a particular time. Therefore, it is not necessarily expected that all timed possessions will provide the same value as a "normal" possession. Also it should be noted that not all timed possessions are necessarily equal in value. For example, if a team is trying to time a shot within 2 seconds, that is very different from trying to time a shot within 23 seconds. The two situations where this paper has shown possession timing strategies exist are the "2-for-1" and end of quarter situations. To understand how different possession length constraints can result in different efficiencies, we can compare how the APPP changes over different possession lengths in these situations.

Figure 7 shows that in both the "2-for-1" and the end of quarter situations, APPP values vary depending on possession length. Although they both show similar fluctuations across time, the "2-for-1" situation is consistently higher in value when compared to an end of quarter situation (especially for shorter possession lengths). A reason for this is because possession flow can be more natural on a "2-for-1" as the time constraint allows for more flexibility. The "2-for-1" strategy does not have to be performed at a particular second (or even at all) whereas the end of quarter situation is bound to a rigid ending time. In a "2-for-1" situation, if a team shoots within the first few seconds of their possession then they are likely doing so because they are expecting a high value in return. Whereas if a team is shooting quickly at the end of the quarter, it is more likely to be a low value desperation shot since there is no value in holding possession as time expires. Lastly, another reason for the lower value is that during an end of quarter possession the defense is aware of the intent of an offense's strategy. Therefore it is possible for the defense to defend differently which can make an end of quarter possession even less valuable. This is unlike the "2-for-1" shot which can come as more of a surprise to the defense as there is a larger window where a shot could occur.

[15] This could be different across different quarters such as the 1st and 3rd where players are more rested and game plans are fresh in players' minds.

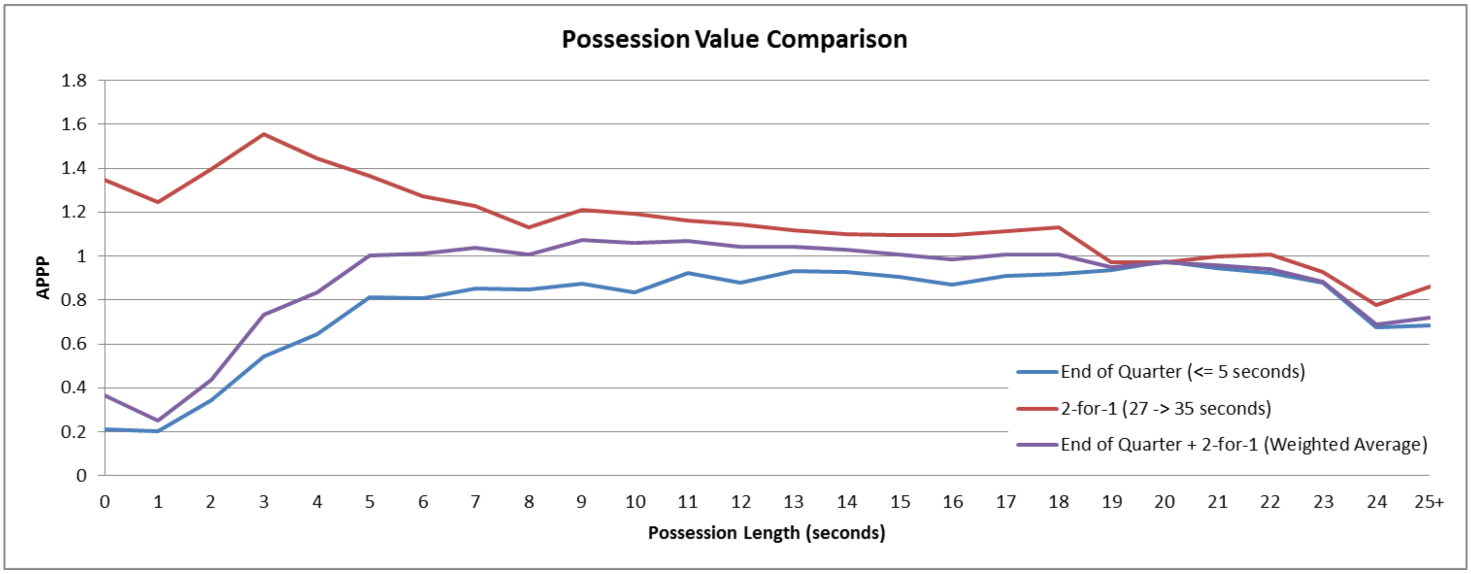

Figure 7. Possession Value Comparison

Timed Possession Value

The intent of building expected "timed possession values" is to be able to apply them to less rigid time frames similar to the existing "2-for-1," such as a "3-for-2". The models which will be introduced in the next section will use these expected values with the intent of allowing flexibility into how strict the timing may be performed. Therefore, it is assumed that in a worst case the value of a timed possession will be equal to the value of an end of quarter possession. In a more optimistic case, the value can be assumed to be closer to that of a "2-for-1" possession. This paper will use the weighted average between the two cases which can be in Figure 7 as calculated using 35,998 samples.

To quantify timed possession value, a new concept called estimated points per possession (EPPP) is introduced. To calculate EPPP the APPP weighted average plot is split into four pieces which will be defined as the following types of possessions:

- **Desperation:** (0-1 seconds) - A "desperation" possession provides the lowest expected value as it is very hard to get a good possession in under 2 seconds when that is the only option. The value of a timed possession for this length is much lower than a "normal" possession, since the non-timed possessions will be opportunistic rather than forced.

- **Rushed:** (2-5 seconds) - A "rushed" possession can be thought of as a possession which has enough time to get a non-desperation shot but it may or may not be of good quality. This type is observed to increase in value linearly with time.

- **Timed:** (6-21 seconds) - A "timed" possession appears to have almost equal value regardless of time used with the slightest decline in value the longer the possession lasts. This consistency can be explained by players running the clock down to a certain point before attacking when trying to intentionally time a possession. This makes any extra time beyond that point irrelevant since it provides minimal impact on value, therefore providing a stable value regardless of time.

- **Timed Desperation:** (22-24 seconds) - A "timed desperation" possession is similar to that of an end of quarter situation as the possession is going to be forced due to the shot clock time constraint. As previously mentioned, the defense is going to know what is coming which will result in a lower value the closer you get to the shot clock expiring.

These four time possession types were chosen because they are intuitive and also translate into a reasonable linear trend lines as shown in Figure 8. The different timed possession values can be compared with the value of a "normal possession." The "normal possession" trend line is calculated using 1,562,761 samples of possessions ending prior to the last minute.

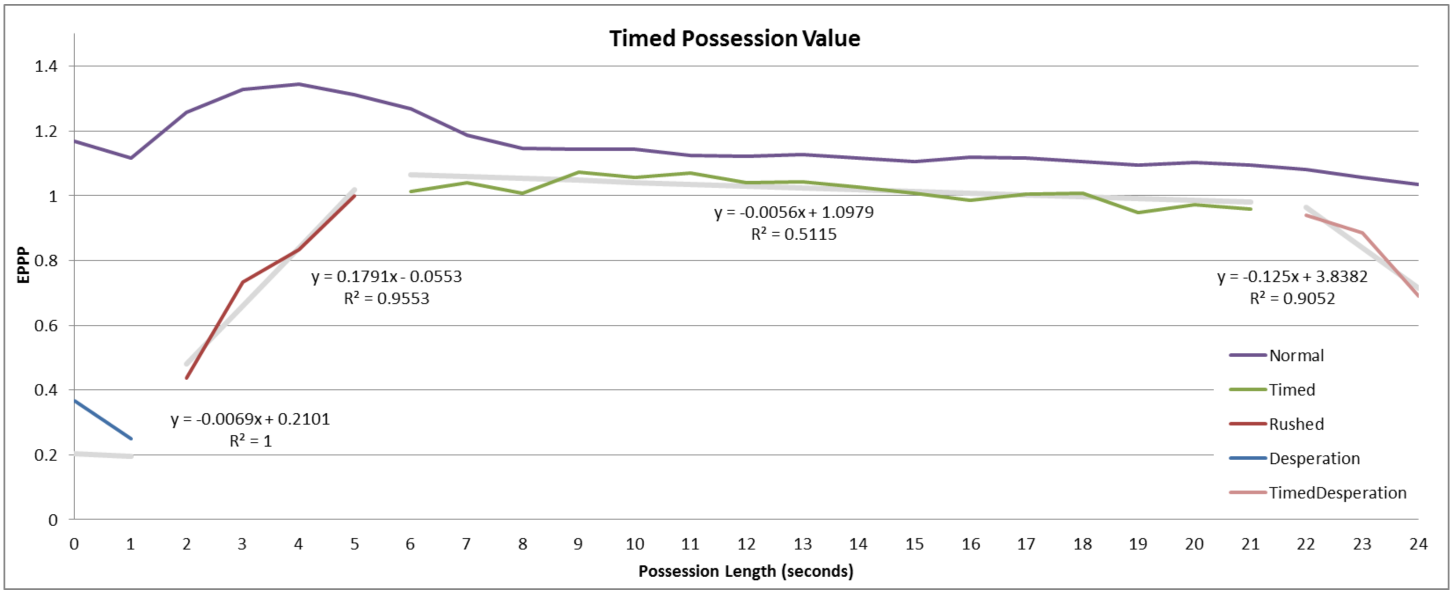

Figure 8. Timed Possession Value

Optimal Possession Timing

Four different methods for evaluating an optimal possession timing strategy are discussed below.

1. NPD Method

By combining NPD (the value of ending a possession at a particular time) and EPPP (the value of a particular possession length) it is possible to determine the optimal possession timing for any given situation. Starting from a possession start time (**S**), there are 25 different possible possession end times before the shot clock expires[16]. Each of those possession end times has an associated NPD (Figure 2) and EPPP value (Figure 8). These values can be represented by the vectors:

$$\boldsymbol{NPD} = [N_{S-0}, N_{S-1}, \dots, N_{S-24}]$$
$$\boldsymbol{EPPP\ Timed} = [T_0, T_1, \dots, T_{24}]$$

Element-wise multiplication is done between the two vectors to calculate the value of each possible timing option:

$$\textbf{Timed Total Value} = \boldsymbol{NPD\ x\ EPPP\ Timed} = [N_{S-0} * T_0, \quad N_{S-1} * T_1, \quad \dots, \quad N_{S-24} * T_{24}]$$

This will produce 25 different "timed possession" options to choose from for a given start time. On top of those choices, a team can also choose to use "normal" possession behavior. The total value of a "normal" possession can be broken up into two pieces: the value of the possession itself and the future NPD value. To estimate the value of the current possession, element-wise multiplication is done on the vectors representing the possession length distribution (Figure 3) and Normal EPPP value (Figure 8):

$$\boldsymbol{EPPP\ Normal} = [E_0, E_1, \dots, E_{24}]$$
$$\boldsymbol{Normal\ Length} = [L_0, L_1, \dots, L_{24}]$$
$$\textbf{Normal Value} = \boldsymbol{EPPP\ Normal\ x\ Normal\ Length} = [E_0 * L_0, \quad E_1 * L_1, \quad \dots, \quad E_{24} * L_{24}]$$

The EPPP values are scaled by the probability of the EPPP values occurring. Similarly this can be done on the NPD values:

$$\textbf{Normal NPD Value} = \boldsymbol{NPD\ x\ Normal\ Length} = [N_{S-0} * L_0, \quad N_{S-1} * L_1, \quad \dots, \quad N_{S-24} * L_{24}]$$

To calculate the total value of a "normal" possession, both vectors are summed:

$$\textbf{Normal Total Value} = \sum_{i=0}^{24} \boldsymbol{Normal\ Value}\ (i) + \sum_{i=0}^{24} \boldsymbol{Normal\ NPD\ Value}\ (i)$$

[16] A possession can use less than a full second all the way up to a full 24 seconds. This research is not taking into account the possibilities of extending the shot clock from a foul, offensive rebound, etc.

By comparing the max possible value from the **Timed Total Value**[17] vector and the single value of **Normal Total Value** it is seen whether or not it is more optimal to intentionally play for a "timed" possession or instead choose to perform a "normal" possession. An advantage of the NPD Method is that incorporates a lot of complicated factors such as: timeouts, fouling, offensive rebounds, turnovers, etc., and it is also easier to think about ("shoot at this time because it has led to a better NPD historically"). A downside of this approach is that it offers no explanation as to "why" it might be a good idea to end a possession at a particular time. In addition, the NPD method doesn't look beyond optimizing the current possession. For example, it will not consider optimizing subsequent possessions together (i.e. it may be optimal to try for a quick shot on the current possession if you know that you can try for another quick shot on the next possession). Figure 9 shows the comparison in value between using a "normal" possession and a "timed" possession at various possession start times[18]. As shown, the NPD method indicates a "2-for-1" benefit up to 0.11 points by using a "timed" possession as opposed to a "normal" possession. However it does indicate a "3-for-2" opportunity. This could be due to its limitations in considering future possession optimization.

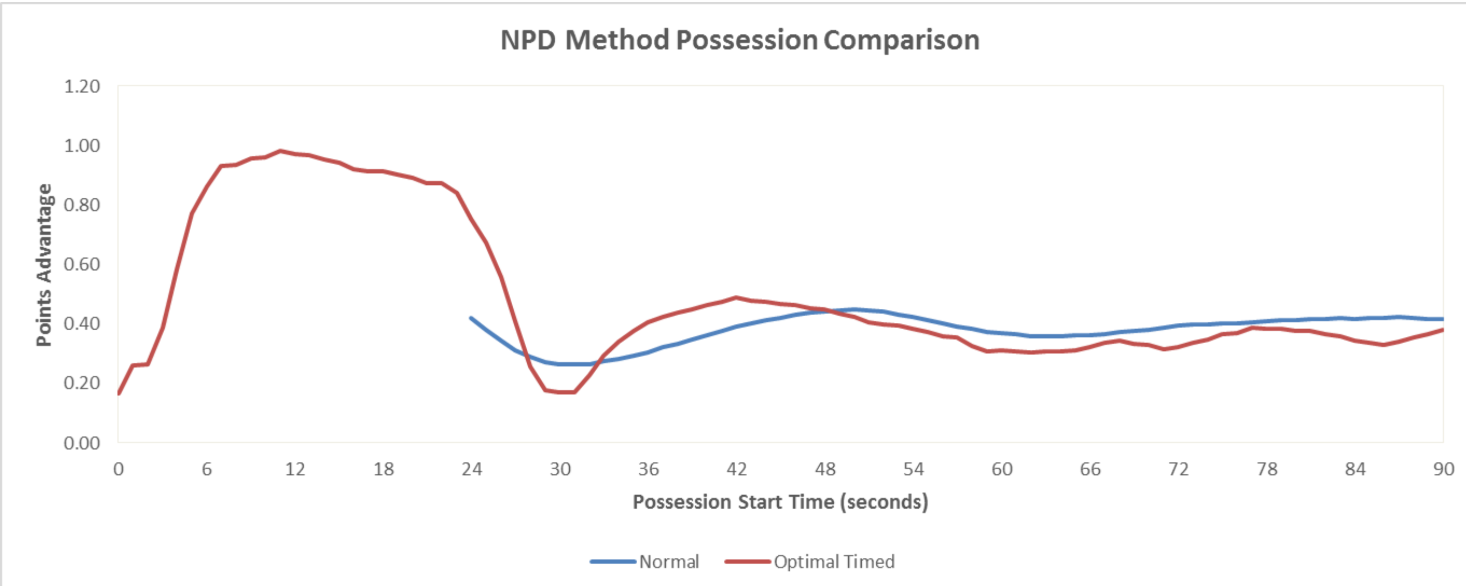

Figure 9. NPD Method Possession Comparison

2. Probabilistic Method

To gain more understanding about why the NPD Method suggests a particular strategy over another, it is useful to see how future possessions play out. By using historical possession timing distributions (**Normal Length**) and associated EPPP values[19], the remainder of the quarter is simulated based upon the probability of the timing of the opponent's next possession. To decide on an optimal strategy, decisions are optimized using the opponent's probabilistic counter move and result (as well as for all subsequent possessions). Rather than using NPD, future value can be simulated by weighting the associated EPPP values using the possession length probabilities. Using this Probabilistic Method it is easier to see how the remainder of the quarter is likely to unfold, which gives a better understanding as to why it is a good idea to time a possession a particular way.

This method assumes that each possession is independent of one another, which in reality may not necessarily be true. A relationship exists between a team's decision in one possession and the response in the subsequent possession. This can be observed when thinking about when a team intentionally tries to rush a possession. This research has observed that rushed possessions have a higher likelihood of both missed shots and turnovers. Both of these are more likely to be followed by a possession with shorter length and higher EPPP value. This is likely because missed shots and turnovers can more easily lead to a fast break opportunities which tend to be quick and high value possessions. Therefore, a drawback of the Probabilistic Method is that it ignores some potentially important interdependencies between possessions.

3. Machine Learning Method

In an attempt to deal with the relationship between one team's possession and its impact on the opponent's subsequent possession, attributes about the current possession can be used to adjust value and expected behavior of a subsequent possession. Similarly, when evaluating the value of certain decisions in the current possession the prior possession's results can be used to influence value and realistic behavior. To

[17] Since it is difficult to "time" a possession on the exact second the results are smoothed using a 3 period simple moving average which essentially implies that when timing a shot the team must try to time their possession within a 3 second window.

[18] There is no "normal" possession data with less than 24 seconds as a "normal" possession formula assumes the ability for a possession to end anytime in the next 24 seconds.

[19] The EPPP values are calculated for a given possession length by including all data, data which this research has defined as "normal", data for the particular possession start time, or some combination. This research assumes data for a particular start time which has the benefit of taking into account real behavior differences at various points but with the downside of potentially smaller sample sizes.

generate such models, machine learning can be used. This problem can be modeled as two separate models: (1) one model which simulates the length of the current possession (as a probability distribution); and (2) a second model which simulates the expected value given an expected possession length. To calculate the optimal strategy you could then recursively simulate both (1) and (2) through the end of the quarter, weighting the likelihood of each outcome according to the probabilities at each step. The probability distributions and associated expected values produced are then combined to give a final estimate of value given a particular decision made in the current possession.

The first model is represented as a multiclass classification problem, the second as a regression problem. Some common machine learning algorithms to use for these two particular representations are random forest machine-learning and linear regression (AIC) respectively. In both models the following features can be used: time remaining in seconds, previous possession type (shot, free throw, turnover, other), and previous possession points. Additionally, the second model has the expected possession length. Both models can be further improved with more features such as: team statistics (pace, offensive/defensive rating, etc.) and player understanding (who is on the court, skill types, playing style).

The advantage of using machine learning for such a method is that you do not have to manually "learn" what factors influence EPPP or possession timing distributions. The models can learn these things for any situation and will understand how behavior and value change at different points in the game. A downside of this approach is that it relies on historical behavior which may not always be consistent going forward as the game continues to evolve.

4. Game Theory Method

Game theory can be defined as: "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" [7]. Rather than relying on historical behavior of teams which may have been acting sub-optimally over the previous 8 years, instead it is more ideal to evaluate a world in which all teams are considering this research and their behavior is dictated accordingly. Using game theory one can simulate the opponents intelligence on when to time their possessions and then model their decisions as such. Instead of optimizing based on historical results (like the NPD Method) or using historical behavior to simulate expected behavior (such as the Probabilistic or Machine Learning Methods), in game theory one would assume the opponent is going to make only optimal decisions and their decisions would be based on that. By working backwards from the end of the quarter it is possible to build the "optimal" strategy for any possible possession start time. This is the exact same methodology as the NPD Method, except that NPD is not used for future possession value and instead the future value is calculated based on assumed subsequent optimal behavior. This value will be called **Optimal Value**. The base case for this process starts with a possession ending on time zero with an expected value of zero:

$$\boldsymbol{Optimal\,Value}[\boldsymbol{S}] = \begin{cases} \mathbf{0} & \boldsymbol{if\ S = 0}; \\ \boldsymbol{max}\left(\sum\limits_{\substack{1 \le = m \le = 24 \\ 0 \le = S - m}} \boldsymbol{T_m} - \boldsymbol{Optimal\,Value}[\boldsymbol{S - m}] \right) & \boldsymbol{otherwise} \end{cases}$$

Using the Game Theory Method, a comparison in value between a "normal" and "timed" possession are shown in Figure 10. Similar to the NPD Method results there is an obvious "2-for-1" time frame. The largest benefit of using a "timed" possession appears for possessions starting at 34 seconds where the expected value exceeds a "normal" possession by 0.29 points. Unlike the NPD Method, the Game Theory Method suggests there is in fact a "3-for-2" opportunity with a max difference in value of 0.06 points for possession starting with 64 seconds remaining. The difference in results is likely because Game Theory Method optimizes all decisions going forward. It is worth noting that the Game Theory Method assumes optimal strategy by the opponent, so the actual opportunity could be greater in practice if not all teams are using such a strategy.

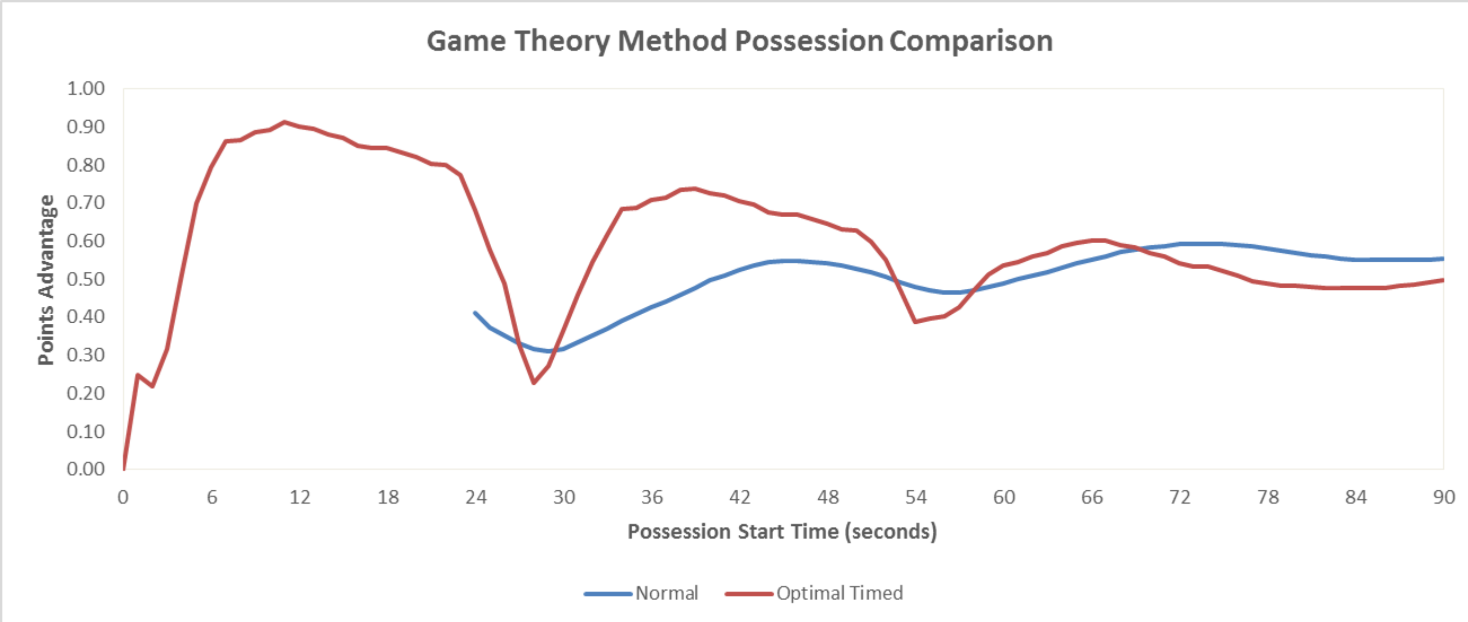

Figure 10. Game Theory Method Possession Comparison

Conclusions

In conclusion this paper has: (1) analyzed the effects that possession timing have on behavior as well as outcome; (2) quantified the cost benefit tradeoff of strategically "timing a possession;" and (3) proposed four different methods to evaluate optimal possession timing strategy. Of the proposed techniques, NPD and Game Theory methods were further pursued and both indicated that even when properly accounting for the reduced value of a "timed" possession, there still exists opportunities for implementing strategic timing. Interestingly, these optimal timing strategies revealed benefits outside of the traditional "2-for-1" time period, which suggests that there is a legitimate "3-for-2" strategy opportunity. A strategy like this will only be effective in real-time situations if it is simplified so that players can execute the strategy with little instruction or deep-thought. To help with that simplification a cheat sheet is proposed in Table 1 which outlines sample results from the Game Theory method evaluation. These advantages could be greatly improved upon if teams were able to increase expected value in "timed" situations whether through coaching, optimal lineups, or extra practice on such situations. Lastly, this same kind of research can be applied to the sports community as a whole, as there could be opportunities for competitive advantages to be gained in other time constrained sports by using similar timing based strategies.

Table 1. Simple Optimal Possession Timing Cheat Sheet

Possession Start Time (seconds)	Simple Strategy	Optimal End Time	Point Advantage Over "Normal" Possession
30 -> 40	Push pace	25 -> 29	0.29
47 -> 52	Slow pace	25 -> 29	0.11
58 -> 66	Push pace	51 -> 56	0.06

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